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SUGGESTED SOLUTION

SYJC

SUBJECT- MATHEMATICS & STATISTICS

Test Code – SYJ 6122

BRANCH - () (Date :)

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SECTION – I

ANSWER : 1

(i) (a) $(p \wedge q) \wedge c$
(b) Dhanashree is a doctor or she is clever

(ii) $|A| = \begin{vmatrix} 1 & 3 & 2 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$

$\therefore\ |A| = -12 \neq 0$

$\therefore$ A is nonsingular. Hence A^{-1} exists.

(iii) $y = (5x - 3)^x$

$\therefore\ \log y = \log (5x - 3)^x$

$\therefore\ \log y = x \log (5x - 3)$

$\therefore \frac{d}{dx}(log y) = \frac{d}{dx} x\ log\ (5x - 3)$

$\therefore\ \frac{1}{y}\ \frac{dy}{dx} = x \frac{d}{dx} \log(5x-3) + log\ (5x - 3)\ \frac{d(x)}{dx}$

$= \frac{x}{5x-3}\ \frac{d}{dx}(5x-3) + \log(5x-3)$

$\therefore\ \frac{1}{y}\ \frac{dy}{dx} = \frac{5x}{5x-3} + \log(5x-3)$

$\therefore\ \frac{dy}{dx} = y\left[\frac{5x}{5x-3} + \log(5x-3)\right]$

$\therefore\ \frac{dy}{dx} = (5x-3)^x\left[\frac{5x}{5x-3} + \log(5x-3)\right]$

(iv) Put $cos\ x = t$ $\quad \therefore -sin\ x\ dx = dt$

$\therefore\ sin\ x\ dx = -dt$

$$\int \frac{\sin x}{1 + cos^2 x} dx = \int \frac{1}{1 + t^2}(-dt)$$

$= -\int \frac{1}{1+t^2} dt = -tan^{-1} t + c$

$= -\tan^{-1}(\cos x) + c.$

(v) $g(0) = e^{\frac{5}{2}}$ (Given) ....(1)

$$\lim_{x \to 0} g(x) = \lim_{x \to 0}\left(1 + \frac{5x}{2}\right)^{\frac{2}{x}}$$

$= \lim_{x \to 0} \left\{ \left(1 + \frac{5x}{2}\right)^{\frac{2}{5x}} \right\}^5$

$= \left\{ \lim_{x \to 0} \left(1 + \frac{5x}{2}\right)^{\frac{2}{5x}} \right\}^5$

$= e^5$ .... [x $\to 0$, $\frac{5x}{2} \to 0$ and $\lim_{x \to 0} (1+\alpha)^{\frac{1}{\alpha}} = e$)

$\therefore \lim_{x \to 0}$ g(x) = e^5

From (1) and (2),

$\lim_{x \to 0}$ g(x) $\neq$ g(0)

$\therefore$ g(x) is discontinuous at x = 0.

(vi) Given f(x) is continuous at x = 2

$\therefore \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$

$\therefore \lim_{x \to 2} \frac{x^2+5}{x-1} = \lim_{x \to 2} (kx+1)$

$\therefore \frac{4+5}{2-1} = k(2)+1$

$\therefore$ 9 = 2k + 1

$\therefore$ k = 4

(vii) The cost function is given as

C = 100 + 600x – $3x^2$

$\therefore \frac{dC}{dx} = \frac{d}{dx}(100 + 600x - 3x^2)$

= 0 + 600 $\times$ 1 – 3 $\times$ 2x

= 600 – 6x

If the total cost is decreasing, then $\frac{dC}{dx} < 0$

$\therefore$ 600 – 6x < 0

$\therefore$ 600 < 6x

$\therefore$ x > 100

Hence, the total cost is decreasing for x > 100.

(viii) $\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

= $\begin{bmatrix} 2+6 & 4+9 & 6+3 \\ 1-4 & 2-6 & 3-2 \end{bmatrix}$

= $\begin{bmatrix} 8 & 13 & 9 \\ -3 & -4 & 1 \end{bmatrix}$

ANSWER : 2(A)

(i)

p	q	~q	p∧ ~q	P → q	(p ∧ ~q) ↔ (p → q)
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	F	T	F

All the entries in the last column of the above truth table are F.

∴ (p ∧ ~q) ↔ (p → q) is a contradiction.

(ii) $x^7x^9 = (x + y)^{16}$

∴ $\log (x^7 y^9) = \log (x + y)^{16}$

∴ $\log x^7 + \log y^9 = \log (x + y)^{16}$

∴ $7 \log x + 9 \log y = 16 \log (x + y)$

Differentiating both sides w.r.t. x, we get,

$7 \times \frac{1}{x} + 9 \times \frac{1}{y}\frac{dy}{dx} = 16 \times \frac{1}{x+y} \cdot \frac{d}{dx}(x+y)$

$\therefore \frac{7}{x} + \frac{9}{y}\frac{dy}{dx} = \frac{16}{x+y}\left(1 + \frac{dy}{dx}\right)$

$\therefore \frac{7}{x} + \frac{9}{y}\frac{dy}{dx} = \frac{16}{x+y} + \frac{16}{x+y} \cdot \frac{dy}{dx}$

$\therefore \left(\frac{9}{y} - \frac{16}{x+y}\right)\frac{dy}{dx} = \frac{16}{x+y} - \frac{7}{x}$

$\therefore \left[\frac{9x + 9y - 16y}{y\,(x+y)}\right]\frac{dy}{dx} = \frac{16x - 7x - 7y}{x\,(x+y)}$

$\therefore \left[\frac{9x-7y}{y(x+y)}\right]\frac{dy}{dx} = \frac{9x-7y}{x(x+y)}$

$\therefore \frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x}$

$\therefore \frac{dy}{dx} = \frac{y}{x}.$

(iii) $\int x^2 \cdot e^{3x}\, dx$

Let I = $\int x^2 \cdot e^{3x}\, dx$

= $x^2 \cdot \int e^{3x} dx - \int \left[\frac{d}{dx} x^2 \int e^{3x} dx\right]\, dx$

= $x^2 \frac{e^{3x}}{3} - \int \left[2x.\frac{e^{3x}}{3}\right] dx$

= $\frac{1}{3} x^2 . e^{3x} - \frac{2}{3} \int x . e^{3x}\, dx$

= $\frac{1}{3} x^2 . e^{3x} - \frac{2}{3} \left\{x . \int e^{3x} dx - \int \left[\frac{d}{dx}(x) \int e^{3x} dx\right] dx\right\}$

= $\frac{1}{3} x^2 . e^{3x} - \frac{2}{3} \left\{x . \frac{e^{3x}}{3} - \left[\int 1 . \frac{e^{3x}}{3} dx\right]\right\}$

= $\frac{1}{3} x^2 . e^{3x} - \frac{2}{9} x . e^{3x} + \frac{2}{9} . \frac{e^{3x}}{3} + \mathrm{c}$

= $\frac{1}{3} x^2 . e^{3x} - \frac{2}{9} x . e^{3x} + \frac{2}{27} . e^{3x} + c$

ANSWER : 2(B)

(i) Let the number of benches sold be x.

Then profit = S.P. – C.P.

i.e., P(x) = $\left(15 - \frac{x}{2000}\right) x - \left(200 + \frac{x}{5}\right)$

= 15x $- \frac{x^2}{2000} - 200 - \frac{x}{5}$

$\therefore$ P(x) = $\frac{74x}{5} - \frac{x^2}{2000} - 200$

$\therefore$ P’(x) = $\frac{d}{dx}\left(\frac{74x}{5} - \frac{x^2}{2000} - 200\right)$

= $\frac{74}{5} \times 1 - \frac{1}{2000} \times 2x - 0$

= $\frac{74}{5} - \frac{x}{1000}$

and P’’(x) = $\frac{d}{dx}\left(\frac{74}{5} - \frac{x}{1000}\right)$

= $0 - \frac{1}{1000} \times 1 = -\frac{1}{1000}$

P’(x) = 0 gives $\frac{74}{5} - \frac{x}{1000} = 0$

$\therefore \frac{x}{1000} = \frac{74}{5}$

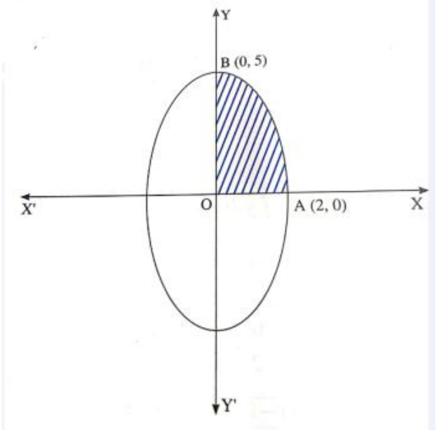
$\therefore$ x = $\frac{74 \times 1000}{5} = 14800$

and P'' (14800) = $-\frac{1}{1000} < 0$

$\therefore$ P(x) is maximum when x = 14800.

Hence, the number of benches sold for maximum profit is 14800.

(ii)



By the symmetry of the ellipse, its area is equal to 4 times the area of the region OABO. Clearly for this region, the limits of integration are 0 and 2.

$\frac{y^2}{25} = 1 - \frac{x^2}{4} = \frac{4-x^2}{4}$

$\therefore$ $y^2 = \frac{25}{4}(4 - x^2)$

In the first quadrant, y > 0

$\therefore$ $y = \frac{5}{2}\sqrt{4 - x^2}$

$\therefore$ area of ellipse = 4(area of the region OABO)

$= 4\int_0^2 y\,dx$

$= 4\int_0^2 \frac{5}{2}\sqrt{4-x^2}\,dx$

$= 10\int_0^2 \sqrt{4-x^2}\,dx$

$= 10\left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$

$= 10\left[\left\{\frac{2}{2}\sqrt{4-4} + 2\sin^{-1}(1)\right\} - \left\{\frac{0}{2}\sqrt{4-0} + 2\sin^{-1}(0)\right\}\right]$

$= 10 \times 2 \times \frac{\pi}{2} = 10\pi$ sq units.

(iii) f(x) = 3x + 2, $2 \leq x \leq 4$

$\therefore$ f(2) = 3(2) + 2 = 8

f(x) = $x^2 + ax + b$, x < 2

$\therefore \lim_{x \to 2^-} f(x) = \lim_{x \to 2}(x^2 + ax + b)$

= $2^2 + a(2) + b = 4 + 2a + b$

Since f is continuous at x = 2, $\lim_{x \to 2^-} f(x) = f(2)$

$\therefore$ 4 + 2a + b = 8

$\therefore$ 2a + b = 4 ....(1)

f(4) = 3(4) + 2 = 14

Also, f(x) = 2ax + 5b, 4 < x

$\therefore \lim_{x \to 4^+} f(x) = \lim_{x \to 4}(2ax + 5b)$

= 2a(4) + 5(b) = 8a + 5b

Since f is continuous at x = 4,
$\lim_{x \to 4^+} f(x) = f(4)$

$\therefore$ 8a + 5b = 14 ....(2)

Multiplying equation (1) by 5, we get,

10a + 5b = 20

Subtracting equation (2) from this equation, we get,

2a = 6 $\therefore$ a = 3

$\therefore$ from (1), 2(3) + b = 4

$\therefore$ 6 + b = 4 $\therefore$ b = – 2

Hence, a = 3 and b = – 2.

ANSWER : 3(A)

(i) u = 1 + sin θ, v = θ – cos θ

Differentiating u and v w.r.t. θ, we get,

$\frac{du}{d\theta} = \frac{d}{d\theta}(1 + \sin\theta) = 0 + cos\ \theta$

= cos θ

and $\frac{dv}{d\theta} = \frac{d}{d\theta}(\theta - \cos\theta) = 1 - (-sin\ \theta)$

= 1 + sin θ

$\therefore \frac{du}{dv} = \frac{(du/d\theta)}{(dv/d\theta)} = \frac{\cos\theta}{1 + sin\theta}$

$\therefore \left(\frac{du}{dv}\right)_{at\ \theta = \frac{\pi}{4}} = \frac{cos\frac{\pi}{4}}{1+\sin\frac{\pi}{4}}$

$= \frac{\left(\frac{1}{\sqrt{2}}\right)}{1+\frac{1}{\sqrt{2}}} = \frac{\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)}$

$= \frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$= \frac{\sqrt{2}-1}{2-1} = \sqrt{2} - 1$

(ii) $\int \sqrt{x^2 + 2x + 5}\, dx$

$= \int \sqrt{x^2 + 2x + 1 - 1 + 5}\ dx$

$= \int \sqrt{(x+1)^2 + 4}\ dx$

$= \int \sqrt{(x+1)^2 + 2^2}\, dx$

$= \frac{1}{2}(x+1)\sqrt{(x+1)^2 + 2^2} + \frac{1}{2}(2^2)\log\left|(x+1) + \sqrt{x^2 + 2x + 5}\right| + c$

$= \frac{(x+1)}{2}\sqrt{x^2 + 2x + 5} + 2\log\left|(x+1) + \sqrt{x^2 + 2x + 5}\right| + c$

(iii) The negation of (~ p ∧ ~ q) ∨ (p ∧ ~ q) is

~ [(~p ∧ ~ q) ∨ (p ∧ ~ q)]

≡ ~ (~ p ∧ ~ q) ∧ ~ (p ∧ ~ q) (Negation of disjunction)

≡ [~ (~ p) ∨ ~ (~ q)] ∧ [~ p ∨ ~ (~q)]

(Negation of conjunction)

≡ (p ∨ q) ∧ (~ p ∨ q) (Negation of negation)

ANSWER : 3(B)

(i) Let I = $\int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}} = \int_0^{\pi/2} \frac{dx}{1+\sqrt{\frac{\sin x}{\cos x}}}$

$\therefore$ I = $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$ ..... (i)

$\therefore$ I = $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}dx$

= $\left\{\int_0^a f(x)dx = \int_0^a f(a-x)dx\right\}$

I = $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ ....(ii)

Adding(i) & (ii) we get,

2I = $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$

= $\int_0^{\frac{\pi}{2}} 1 \;\; dx$

= $[x]_0^{\frac{\pi}{2}}$

= $\frac{\pi}{2}-0$

$\therefore$ 2I = $\frac{\pi}{2}$

$\therefore$ I = $\frac{\pi}{4}$

(ii) The given equations can be considered in the matrix equation as

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

i.e. AX = B ....(i)

Now A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$

Here, $|A|$ = 10 $\neq$ 0 $\therefore$ A^{-1} exists

Pre multiplying equation (i) by A^{-1} we get

$(A^{-1}A)X = A^{-1}B$

i.e. $IX = A^{-1}B$

i.e. $X = A^{-1}B$ .....(ii)

Now as A^{-1} exists, consider the equation

$$AA^{-1} = I$$

i.e. $\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix} A^{-1} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

Use $R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - R_1$

$\therefore \begin{bmatrix}1 & -1 & 1\\ 0 & 3 & -5\\ 0 & 2 & 0\end{bmatrix} A^{-1} = \begin{bmatrix}1 & 0 & 0\\ -2 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}$

Use $R_2 \to R_2 - R_3$

$\therefore \begin{bmatrix}1 & -1 & 1\\ 0 & 1 & -5\\ 0 & 2 & 0\end{bmatrix} A^{-1} = \begin{bmatrix}1 & 0 & 0\\ -1 & 1 & -1\\ -1 & 0 & 1\end{bmatrix}$

Use $R_1 \to R_1 + R_2$

and $R_3 \to R_3 - 2R_2$

$\therefore \begin{bmatrix}1 & 0 & -4\\ 0 & 1 & -5\\ 0 & 0 & 10\end{bmatrix} A^{-1} = \begin{bmatrix}0 & 1 & -1\\ -1 & 1 & -1\\ 1 & -2 & 3\end{bmatrix}$

Use $\frac{1}{10}R_3$

$\therefore \begin{bmatrix}1 & 0 & -4\\ 0 & 1 & -5\\ 0 & 0 & 1\end{bmatrix} A^{-1} = \begin{bmatrix}0 & 1 & -1\\ -1 & 1 & -1\\ \frac{1}{10} & \frac{-2}{10} & \frac{3}{10}\end{bmatrix}$

Use $R_1 \to R_1 + 4R_3$

$R_2 \to R_2 + 5R_3$

$\therefore \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} A^{-1} = = \begin{bmatrix}\frac{4}{10} & \frac{2}{10} & \frac{2}{10}\\ \frac{-5}{10} & 0 & \frac{5}{10}\\ \frac{1}{10} & \frac{-2}{10} & \frac{3}{10}\end{bmatrix}$

i.e. $= \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} A^{-1} = \frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}$

$A^{-1} = \frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}$

Note : A^{-1}can also be found by adjoint method.

Using equation (ii) we get

$$X = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16+4 \\ -20+10 \\ 4+6 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Hence, using equality of matrices, we get x = 2, y = – 1, z = 1, which is the required solution of the system of given equations.

(iii) **(a)** Elasticity of demand

$$\eta = \frac{-p}{x} \cdot \frac{dx}{dp}$$

For $x = 200 - 4p$,

$$\frac{dx}{dp} = -4$$

$$\therefore \eta = \frac{-p}{x} \cdot \frac{dx}{dp}$$

$$= \frac{-p}{(200-4p)}(-4) \qquad \text{(For } p < 50\text{)}$$

$$\therefore \eta = \frac{p}{(50-p)} \qquad \text{(For } p < 50\text{)}$$

(b) When p = 10

$$\eta = \frac{10}{(50-10)}$$

$$= \frac{10}{40}$$

$$= 0.25 < 1$$

$\therefore$ Demand is relatively inelastic for p = 10

When p = 30

$$\eta = \frac{30}{(50-30)}$$

$$\eta = \frac{30}{20}$$

$$= 1.5 > 1$$

$\therefore$ Demand is relatively elastic when p = 30

(c) To find the price when η = 1

As η = 1,

$$\therefore \frac{p}{50-p} = 1$$

$$\therefore p = 50 - p$$

$\therefore$ 2p = 50

$\therefore$ p = 25

$\therefore$ For elasticity η equal to 1, price is 25/ unit.

SECTION – II

ANSWER : 4

(i) Given, b_{yx} = – 0.4, bxy = – 2.025, r = ? (1/2 marks)

Correlation coefficient :

r = $\pm\sqrt{b_{yx} \times b_{xy}}$ (1/2 marks)

= $\pm\sqrt{(-0.4) \times (-2.025)}$

= $\pm\sqrt{0.81}$

r = – 0.9 $\because$ b_{yx} & b_{xy} are negative (1 mark)

(ii) P(X = 1) = $\frac{1}{5}$

P (x = 2) = $\frac{1}{5}$

P (x = 3) = $\frac{1}{5}$

P (x = 4) = $\frac{1}{5}$

½ mark

x = x	P(x = x)	x · p(x)
1	$\frac{1}{5}$	$\frac{1}{5}$
2	$\frac{1}{5}$	$\frac{2}{5}$
3	$\frac{1}{5}$	$\frac{3}{5}$
4	$\frac{1}{5}$	$\frac{4}{5}$
	Total	$\frac{10}{5}$

½ mark

$\therefore$ Expected value : E(x)= $\sum x \cdot p(x)$ (1/2 mark)

= $\frac{10}{5}$

$\therefore$ E(X) = 2 (1/2 Mark)

(iii) Given, Present worth (PW) = Rs. 5,500

Sum due (SD) = Rs. 5,830

Period (n) = 9 months

$= \frac{9}{12} = \frac{3}{4}$ years

½ mark

Now, S.D. = PW $\left(1 + \frac{n \times r}{100}\right) \rightarrow$ (1/2 mark)

$5830 = 5500 \left(1 + \frac{3/4\, r}{100}\right) \rightarrow$ (1/2 mark)

$\frac{5830}{5500} = 1 + \frac{3r}{400}$

$\frac{5830}{5500} - 1 = \frac{3r}{400}$

$\frac{5830}{5500} = 1 + \frac{3r}{400}$

$\frac{330}{5500} = \frac{3r}{400}$

$\frac{330}{55} \times \frac{4}{3} = r$

r = 8% p.a. $\rightarrow$ (1/2 mark)

Hence, rate of interest is 8% p.a.

(iv) Given, $\sum d^2 = 66$

n = 10

½ mark

$\therefore$ Rank Correlation :

$R = 1 - \frac{6 \sum d^2}{n(n^2-1)}$ $\rightarrow$ (1/2 mark)

$= 1 - \frac{6 \times 66}{10\,(10^2-1)}$ $\rightarrow$ (1/2 mark)

$= 1 - \frac{6 \times 66}{10\,(100-1)}$

$= 1 - \frac{6 \times 66}{10 \times 99}$

$= 1 - \frac{2}{5}$

$= \frac{3}{5}$

R = 0.6 $\rightarrow$ (1/2 mark)

(v) Anandi invested Rs. 10,000 for 7 months and Rutuja invested Rs. 10,000 for 12 months.

$\therefore$ Profit is distributed in the ratio.

$(10,000 \times 7) : (10,000 \times 12)$

i.e., 70,000 : 1,20,000
i.e., 7 : 12

Also, 7 + 12 = 19

Anandi's share in the profit = $\frac{7}{19} \times 5700$

= Rs. 2,100

$\therefore$ Rutuja's share in the profit = $\frac{12}{19} \times 5700$

= Rs. 3,600

(2)

(vi)

Age group (Years)	**Population**	**No. of Deaths**	**Age – SDR (per thousand)**
	nPx	**nDx**	$= \frac{{}_nD_x}{{}_nP_x} \times \mathbf{1000}$
Below 10	25	50	2 ($\frac{1}{2}$ mark)
10 – 30	30	90	3 ($\frac{1}{2}$ mark)
30 – 45	40	160	4 ($\frac{1}{2}$ mark)
45 – 70	20	100	5 ($\frac{1}{2}$ mark)

(vii) Given, p = Rs. 10,000

r = 10% p.a.

n = 3 years

$\therefore$ i = $\frac{r}{100} = \frac{10}{100}$ = 0.1

½ mark

Using the formula (Relation between A & P)

$A = P(1 + i)^n$ $\rightarrow$ (1/2 mark)

$A = 10,000\,(1 + 0.1)^3$ $\rightarrow$ (1/2 mark)

$= 10000\,(1.1)^3$

$= 10000\,(1.331)$

A = Rs. 13,310 $\rightarrow$ (1/2 mark)

$\therefore$ Accumulated value after 3 years is Rs. 13,310

(viii) Step (1) Row minima :

Subtract the smallest element in each row from every element in that row.

Machines	I	II	III
	Jobs		
M_1	0	3	4
M_2	2	0	5
M_3	4	5	0

(1/2 mark)

Step (2) column minima

Subtract the smallest element in each column from every element in that column

Machines	I	II	III
	Jobs		
M_1	0	3	4
M_2	2	0	5
M_3	4	5	0

(1/2 mark)

Step(3) Cover maximum zeros with minimum no. of straight lines.

Machines	I	II	III
	Jobs		
M_1	~~0~~	~~3~~	~~4~~
M_2	~~2~~	~~0~~	~~5~~
M_3	~~4~~	~~5~~	~~0~~

(1/2 mark)

Since the number of straight line covering all zeros is equal to number or row, the optimum solution has reached.

Step 4 : Assignment
The optimal assignment can be made as follows :

Machines	I	II	III
	Jobs		
M_1	$\boxed{0}$	3	4
M_2	2	$\boxed{0}$	5
M_3	4	5	$\boxed{0}$

(1/2 mark)

Step 5 : Allocation

Hence, the optimum assignment schedule is obtained as follows :

M_1	I	1
M_2	II	2
M_3	III	3

$\therefore$ Minimum value = 6 units.

ANSWER : 5 (A)

(i)

x	**Y**	**(x - $\bar{x}$)**	**(y - $\bar{y}$)**	**(y - $\bar{y})^2$**	**(x - $\bar{x}$)**
21	19	-2	-1	1	2
25	20	2	0	0	0
26	24	3	4	16	12
24	21	1	1	1	1
19	16	-4	-4	16	16
115	**100**			**34**	**31**
				(1/2 mark)	**(1/2 mark)**

Step (i) $\bar{x} = \frac{\sum x}{n} = \frac{115}{5} = 23$

$\bar{y} = \frac{\sum y}{n} = \frac{100}{5} = 20$

Step (ii) Regression coefficient of x on y

$b_{xy} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(y-\bar{y})^2}$ → (1/2 mark)

$= \frac{31}{34}$

= 0.91176

~ 0.9118 → (1/2 mark)

Step (iii) Regression equation of x on y

$(x-\bar{x}) = b_{xy}\,(y-\bar{y})$ → ½ mark

(x – 23) = 0.9118 (y – 20)

x – 23 = 0.9118y – 18.235

∴ y = 0.9118 y – 18.235 + 23

y = 0.9118y + 4.764 → $\frac{1}{2}$ mark

(ii) Let the Banker's Gain be written as BG = Rs. x

Let Banker's Discount be written as BD

Given BD = 26 × BG → ½ mark

i.e. BD = 26 × x

= Rs. 26 x → ½ mark

Now, BG = BD – TD → ½ mark

$x = 26x - TD$

$TD = 26x - x$

$= 25x$ → ½ mark

But B.G. = Interest on TD for 1 year

$= \frac{TD \times n \times r}{100}$ → ½ mark

$x = \frac{25x \times 1 \times r}{100}$

$100x = 25x\, r$

$\therefore$ r = 4% → ½ mark

$\therefore$ Rate of interest is 4% p.a.

(iii) Here, $l_{91} = 97$, $d_{91} = 38$ and $q_{92} = \frac{27}{59}$

Now, $d_{91} = l_{91} - l_{92}$ → (1/2 mark)

$38 = 97 - l_{92}$

$\therefore$ $l_{92} = 97 - 38 = 59$ → (1/2 mark)

Now, $q_{92} = \frac{d_{92}}{l_{92}}$ → (1/2 mark)

$\frac{27}{59} = \frac{d_{92}}{59}$

$\Rightarrow$ $d_{92} = 27$ → (1/2 mark)

$\therefore$ $d_{92} = l_{92} - l_{93}$ → (1/2 mark)

$27 = 59 - l_{93}$

$\therefore$ $l_{93} = 59 - 27 = 32$ → (1/2 mark)

Hence, $l_{92} = 59$ and $l_{93} = 32$

ANSWER : 5 (B)

(i) Consider the equation 2x + y = 4

Putting x = 0, y = 0

$0 + 0 \not\geq 4$

Therefore, the solution set is away from the origin.

	A	B
x	0	2
y	4	0

Consider x + y = 5

Putting x = 0, y = 0

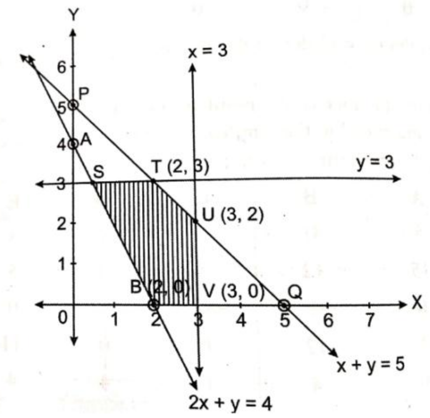
$0 + 0 \leq 5$

$\therefore$ The solution set is towards origin

	P	Q
x	0	5
y	5	0

x = 3 is a line passing through (3, 0) and parallel to Y – axis. Solution set is towards origin.

y = 3 is a line passing through (0, 3) and parallel to X – axis. The solution set is towards origin.



The solution set is a common feasible solution BSTUVB with B(2, 0), V(3, 0).

T is a point of intersection of lines

y = 3 and x + y = 5

Putting y = 3 in x + y = 5, we get

x + 3 = 5

$\therefore$ x = 2

$\therefore$ T(2, 3)

U is a point of intersection of lines x = 3 and x + y = 5

Putting x = 3 in x + y = 5, we get y = 2.

$\therefore$ U (3, 2)

S is a point of intersection of lines y = 3 and 2x + y = 4

Putting y = 3 in 2x + y = 4, we get

$2x + 3 = 4$

$\therefore \quad 2x = 1$

$\therefore \quad x = \frac{1}{2} = 0.5$

$\therefore \quad S = (0.5, 3)$

Now, $\quad Z = 4x + 5y$

$\therefore \quad Z(B) = 4 \times 2 + 5 \times 0 = 8 \qquad [\because B = (2, 0)]$

$\therefore \quad Z(V) = 4 \times 3 + 5 \times 0 = 12 \qquad [\because V = (3, 0)]$

$\therefore \quad Z(T) = 4 \times 2 + 5 \times 3 = 23$ maximum $\qquad [\because T = (2, 3)]$

$\therefore \quad Z(V) = 4 \times 3 + 5 \times 2 = 22 \qquad [\because V = (3, 2)]$

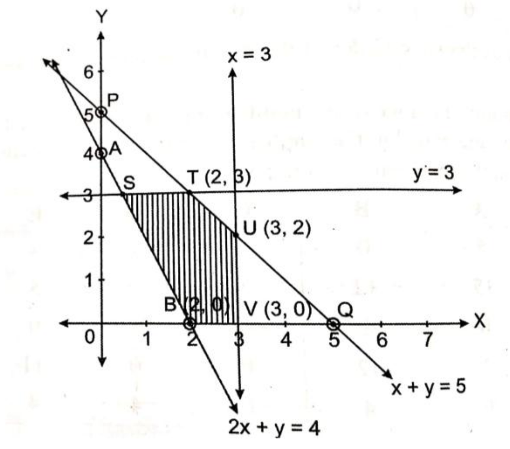
$\therefore \quad Z(S) = 4 \times 0.5 + 5 \times 3 = 17 \qquad [\because S = (0.5, 3)]$

Therefore, maximum value of Z is 23 at x = 2, y = 3.

(4)

Alternative Solutions :

Inequations	**Equations**	**X**	**Y**	**(x , y)**	**Solution set**
$0 \leq x \leq 3$	x = 3	3	0	(3, 0)	0 < 3 (T) Origin side (LHS of line)
$0 \leq y \leq 3$	y = 3	0	3	(0, 3)	0 < 3 (T) origin side (Below of line)
$x + y \leq 5$	x + y = 5	5 0	0 5	(5,0) (0, 5)	0 + 0 < 5 0 < 5 (T) origin side
$2x + y \geq 4$	2x + y = 4	0	4	(0, 4)	2(0) + 0 > 4 0 > 4(F) Non – origin side



From graph, ABCDE be the Feasible region
Here A = (0.5, 3), B (2, 3), C (3, 2) D = (3, 0) & E (2, 0)

$\therefore$ By corner point method

Corners	Objective function Z = 4x + 5y
A (0.5, 3)	Z = 4(0.5) + 5(3) = 2 + 15 = 17
B(2,3)	Z = 4(2) + 5(3) = 8 + 15 = 23
C(3, 2)	Z = 4(3) + 5(2) = 12 + 10 = 22
D (3, 0)	Z = 4(3) + 5(0) = 12 + 0 = 12
E (2, 0)	Z = 4(2) + 5(0) = 8 + 0 = 8

$\therefore Z_{max} = 23$ at x = 2 & y = 3

(ii) Here, Min.(A) = 5, Max. (B) = 5, Min. (C) = 1

Since, Min. (A) $\geq$ Max. (B) is satisfied, the problem can be converted into 5 job 2 machine problem. Two fictitious machines are,

G = A + B, H = B + C.

The problem now can be written as follows :

Jobs	**Machines**	
	G = A + B	**H = B + C**
1	6	2
2	15	9
3	7	4
4	12	8
5	9	7

Hence, Min. (G_{i1}, H_{i2}) = 2, which corresponds to machine H.

Therefore, job 1 is processed in the last.

				1

The problem now reduces to remaining four jobs.

Here, Min. (G_{i1}, H_{i2}) = 4, which corresponds to machine H.

Therefore, job 3 is processes in the last next to job 1.

			3	1

The problem now reduces to remaining three jobs.

Here, Min. (G_{i1}, H_{i2}) = 7, which corresponds to machine H.

		5	3	1

The problem now reduces to two jobs 2 and 4.
Here, Min. (G_{i1}, H_{i2}) = 8, which corresponds to machine H.

Therefore, job 4 is processed in the last next to job 5 and then job 2 is processed first.

Thus, the optimal sequence of jobs is obtained as follows :

2	4	5	3	1

Total elapsed time to complete the tasks can be computed as follows :

Job Sequence	**Machine A**		**Machine B**		**Machine C**		**Idle time for C**
	Time in	**Time out**	**Time in**	**Time out**	**Time in**	**Time out**	
2	0	11	11	15	15	20	15
4	11	18	18	23	23	26	3
5	18	24	24	27	27	31	1
3	24	29	27	29	31	33	0
1	29	34	34	35	35	36	2
						Idle time for machine C	21

From the above table :

Total elapsed time T = 36 hours.

Idle time for machine A

= T – $\begin{Bmatrix} Sum\ of\ procesing\ time \\ of\ all\ jobs\ on\ machine\ A \end{Bmatrix}$

= 36 – 34

= 2 hours.

Idle time for machine B

= T – $\begin{Bmatrix} Sum\ of\ procesing\ time \\ of\ all\ jobs\ on\ machine\ B \end{Bmatrix}$

= 36 – (1 + 4 + 2 + 5 + 3)

= 36 – 15

= 21 hours

Idle time for machine C = 21 hours.

(iii) **Step (i)** $\left.\begin{aligned} \bar{x} &= \frac{\sum x}{n} = \frac{15}{5} = 3 \\ \bar{y} &= \frac{\sum y}{n} = \frac{30}{5} = 6 \end{aligned}\right\}$ 1 mark

Step (ii)

x	y	x^2	y^2	xy
3	8	9	64	24
2	4	4	16	8
1	10	1	100	10
5	2	25	4	10
4	6	16	36	24
		55	**220**	**76**

Table 1 mark

Step (iii) Karl Pearson's correlation coefficient

$$r = \frac{n\sum xy - \sum n \sum y}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2\,(\sum y)^2}}$$

$= \frac{5 \times 76 - 15 \times 30}{\sqrt{5 \times 55 - (15)^2}\sqrt{5 \times 220 - (30)^2}}$ (1 Mark)

$= \frac{380-450}{\sqrt{275-225}\,\sqrt{110-900}}$

$= \frac{-70}{\sqrt{50}\,\sqrt{200}}$

$= \frac{-70}{\sqrt{10000}}$

$= \frac{-70}{100}$

$= -0.7$ (1 mark)

ANSWER : 6 (A)

(i) Let x : no. of answers true

i.e. x = 0, 1, 2, …… ,7

p = Probability of true answer

$= \frac{1}{2}$

½ mark

$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

Given, n = 7 (No. of total questions)

$\therefore\ x \sim B(n, P)$ (1/2 mark)

$P[x = x] = {}^nC_x\, p^x\, q^{h-x}$

Now, $P[X \le 3] = p[x = 0] + p[x = 1] + P[x = 2] + p[x = 3]$ $\rightarrow$ (½ mark)

$= {}^7C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^7 + {}^7C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^6 + {}^7C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^5 + {}^7C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$ → ½ mark

$= \left(\frac{1}{2}\right)^7 [{}^7C_0 + {}^7C_1 + {}^7C_2 + {}^7C_3]$

$= \frac{1}{128}[1 + 7 + 26 + 35]$

$= \frac{69}{128}$ → ½ mark

$= 0.5391$ → ½ mark

(ii) Given, S.D. = Rs. 36,600, n = 4 months, r = 5% → (1/2 mark)

Now, B.D. $= \frac{S.D.\times n \times r}{100} = \frac{36{,}600 \times 1 \times 5}{3 \times 100}$ $\left(\because n = \frac{1}{3} year\right)$

= Rs. 610 → (1/2 mark)

Let T.D. = Rs. x

B. D. = T.D. + Interest on T.D. for $\frac{1}{3}$ year at 5% p.a. (1/2 mark)

$\therefore$ $610 = x + \left(x \times \frac{1}{3} \times \frac{5}{100}\right)$

$\therefore$ $610 = x + \frac{x}{60} = \frac{61x}{60}$

$x = \frac{610 \times 60}{61}$ = Rs. 600 (1/2 mark)

Banker's gain = Banker's discount – True discount →(1/2 mark)

= Rs. (610 – 600)

= Rs. 10

Banker's gain = Rs. 10 → (1/2 mark)

(3)

(iii) Let x_1 = Number of air conditioners

x_2 = Number of fans

Since, the number of products cannot be negative.

$x_1 \geq 0, x_2 \geq 0$

The information given can be represented in tabular form as follows :

Section	Requirement (in hour)		Availability (in hours)
	Air conditioners (x_1)	**Fans (x_2)**	
Wiring	4	2	240

Drilling	2	1	100
Profit Rs.	2000	1000	$Z = 2000x_1 + 1000x_2$

From the above table, we get

$4x_1 + 2x_2 \le 240$

$2x_1 + x_2 \le 100$

and objective function $Z = 2000x_1 + 1000x_2$

Hence, LPP is formulated as follows :

Maximize $Z = 2000x_1 + 1000x_2$

Subject to constraints

$4x_1 + 2x_2 \le 240$

$2x_1 + x_2 \le 100$

$x_1 \ge 0, x_2 \ge 0.$

ANSWER : 6 (B)

(i) We know that, CDR = $\frac{\Sigma D_i}{\Sigma P_i} \times 100$ → (1/2 mark)

For District A :

$$\left.\begin{array}{l}\Sigma D_i = 20 + 30 + 40 = 90 \\ \Sigma P_i = 1000 + 3000 + 2000 = 6000\end{array}\right\}$$ (1/2 mark)

$\therefore$ CDR for district A denoted by CDR_A is

$$CDR_A = \frac{\Sigma D_i}{\Sigma P_i} \times 1000 = \frac{90}{6{,}000} \times 1000 = 15 \text{ per thousand}$$

For District B :

$\Sigma D_i = 50 + 70 + 25 = 145$

$\Sigma P_i = 2000 + 7000 + 1000 = 10{,}000$ (1/2 mark)

$\therefore$ CDR for district B denoted by CDR_B is

$$CDR_B = \frac{\Sigma D_i}{\Sigma P_i} \times 1000 = \frac{145}{10{,}000} \times 1000 = 14.5 \text{ per thousand}$$

Hence, the district B is more healthy as $CDR_B < CDR_A$. (1/2 mark)

(4)

(ii) In Poisson distribution

$x \sim p(m)$

$P[x = x] \quad = \frac{e^{-m}.m^x}{x!}$

Given, P(x = 1) = P(x = 2)

$\frac{e^{-m} \cdot m^1}{1!} = \frac{e^{-m} \cdot m^2}{2!}$

$\frac{m}{1} = \frac{m^2}{2}$

$\frac{2}{1} = \frac{m^2}{m}$

m = 2

Now, $p(x \geq 1) = 1 - p(x < 1)$

$= 1 - p[x = 0]$

$= 1 - \frac{e^{-2} \cdot 2^0}{0!}$

$= 1 - \frac{0.1353 \times 1}{1}$

$= 1 - 0.1353$

$= 0.8647$

(iii) The Value of a car = Rs. 4,00,000

The policy value of a car = Rs. 2,50,000 (0.5 Mark)

The rate of Premium = 5% - (20% of 5)

= 5% - 1%

= 4% (0.5 Mark)

Therefore Premium amount is = $\frac{4}{100} \times 2{,}50{,}000$

= Rs. 10,000/- (0.5 Mark)

If the value of the car is reduced to 60% of its original value,

then the value of the car is = Rs. 4,00,000 X $\frac{60}{100}$ = Rs. 2,40,000/- (0.5 Mark)

Therefore loss is = Rs. (4,00,000 – 2,40,000) = 1,60,000/- (0.5 Mark)

Now, claim is = $\frac{Insured\ Value}{Value\ of\ car} \times loss$ = $\frac{2{,}50{,}000}{4{,}00{,}000} \times 1{,}60{,}000$ = Rs. 1,00,000/- (0.5 Mark)

Next, Net Loss = Loss – Claim

= 1,60,000 – 1,00,000 = 60,000/- (0.5 Mark)

Therefore Loss the owner bear (Including premium) = Net Loss + Premium

= 60,000 + 10,000 = Rs. 70,000 (0.5 Mark)